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Addendum to: "Coupled KdV Equations of Hirota-Satsuma Type" (J. Nonlin. Math. Phys. Vol. 6, No.3 (1999), 255–262)

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Abstract

It is shown that one system of coupled KdV equations, found in *J. Nonlin. Math. Phys.*, 1999, V.6, N 3, 255–262 to possess the Painlevé property, is integrable but not new.

In our recent paper [1], we found that the system of coupled KdV equations

$$
\begin{aligned}
u_t &= u_{xxx} + 9v_{xxx} - 12uu_x - 18vu_x - 18uv_x + 108vv_x,\\
v_t &= u_{xxx} - 11v_{xxx} - 12uu_x + 12vu_x + 42uv_x + 18vv_x
\end{aligned}
\tag{1}
$$

passes the Painlevé test for integrability well, but we were unable to find a parametric zero-curvature representation for this system there. In this addendum, we show that the system (1) *is integrable but not new*: it is related by a simple transformation of variables to an integrable system introduced a long time ago by Drinfeld and Sokolov [2].

In their paper, in Example 13, Drinfeld and Sokolov gave the Lax representation $L_t = [A, L]$ with the differential operators

$$
\begin{aligned}
L &= \left(D^3 + 2uD + u_x\right)\left(D^2 + v\right),\\
A &= D^3 + \left(\frac{6}{5}u + \frac{3}{5}v\right)D + \left(-\frac{3}{5}u_x + \frac{6}{5}v_x\right)
\end{aligned}
\tag{2}
$$

for the system of coupled KdV equations

$$
\begin{aligned}
u_t &= -\frac{4}{5}u_{xxx} + \frac{3}{5}v_{xxx} - \frac{12}{5}uu_x + \frac{3}{5}vu_x + \frac{6}{5}uv_x,\\
v_t &= \frac{3}{5}u_{xxx} - \frac{1}{5}v_{xxx} + \frac{12}{5}vu_x + \frac{6}{5}uv_x - \frac{6}{5}vv_x.
\end{aligned}
\tag{3}
$$

It is easy to see that the transformation

$$
t \to 10t, \qquad u \to -\frac{3}{2}u + \frac{3}{2}v, \qquad v \to -2u - 3v \tag{4}
$$

changes the system (3) into the system (1). This solves the problem of integrability of the system (1). Moreover, using the scalar spectral problem $L\phi = \lambda\phi$, $\phi_t = A\phi$, where the operators L and A are given by (2) and λ is a parameter, and the transformation (4), we can construct the first-order linear problem $\Psi_x = X\Psi$, $\Psi_t = T\Psi$, or the zero-curvature representation $X_t = T_x - [X,T]$, for the system (1), with the following 5×5 matrices X and T:

$$X = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 2u+3v & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{3}{2}(u-v) & 0 & 1 \\ \lambda & 0 & 0 & \frac{3}{2}(u-v) & 0 \end{pmatrix},$$

$T = \{\{5u_x - 15v_x, -10u + 30v, 0, 10, 0\}$, $\{5u_{xx} - 15v_{xx} - 20u^2 + 30uv + 90v^2, -5u_x + 15v_x, 5u+15v, 0, 10\}$, $\{10\lambda, 0, 30v_x, -30v, 0\}$, $\{0, 10\lambda, 30v_{xx} - 45uv + 45v^2, 0, -30v\}$, $\{5\lambda u + 15\lambda v, 0, 10\lambda, 30v_{xx} - 45uv + 45v^2, -30v_x\}\}$, where the cumbersome matrix T is written by rows.

We can conclude now, that *all* the systems of coupled KdV equations, which passed the Painlevé test in [1], have turned out to be integrable.

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References

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[2] Drinfeld V G and Sokolov V V, New Evolutionary Equations Possessing (L-A)-Pair, in Proceedings of the S L Sobolev Seminar, Novosibirsk, 1981, Volume 2, 5–9 (in Russian).